ON IMPEDANCE MATCHING: A DIFFERENT LOOK

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The Matching Theorem states that for maximum power transfer from a signal source to its load, the load impedance must be equal to the complex conjugate of the source impedance, and vice versa, i.e. if $Z_S = R_S + jX_S$, then the load impedance must be $Z_L = R_S + j(-X_S)$ (see Fig. 1). This relation implicitly expresses that the matching is perfect only at a certain frequency, but it does not give any information about what happens at other frequencies. The signal source can be a primary source, for example an antenna, the output of the previous stage of an electronic circuit, or one end of a transmission line. Similarly, the load can be a passive load, for example an antenna, the input of the following stage of an electronic circuit, or one end of a transmission line.

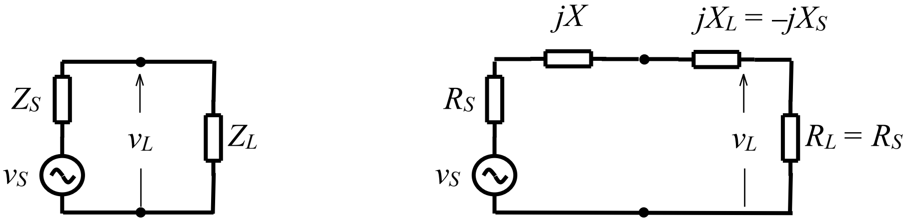
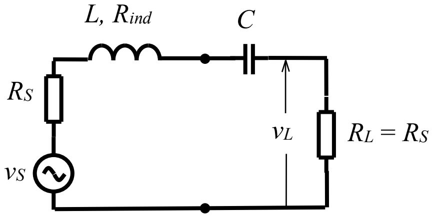


Fig. 1 Classical impedance matching

The Matching Theorem does not provide a direct design hint to the design engineer. But if we interpret this statement from the physics point of view, we see that impedance matching corresponds to the cancellation of the reactive parts with respect to each other, in other words, creating a series resonance circuit consisting of the complex conjugate impedances (see Fig. 2). This indicates that the matching is perfect only at the resonance frequency and has a bandwidth around the resonance frequency, determined by the quality factor of this resonance circuit:

$$Q = \frac{L\omega_0}{R_{tot}}$$



$X_S = \omega_0 L$

$X_L = -\omega_0 L = -1/C\omega_0$

Fig. 2 Impedance matching from the point of view of series resonance

Since the value of the total series resistance of the resonance circuit is $R_{tot} = 2R_S + R_{ind} \cong 2R_S$, which is usually high, the quality factor is very low. For low-Q

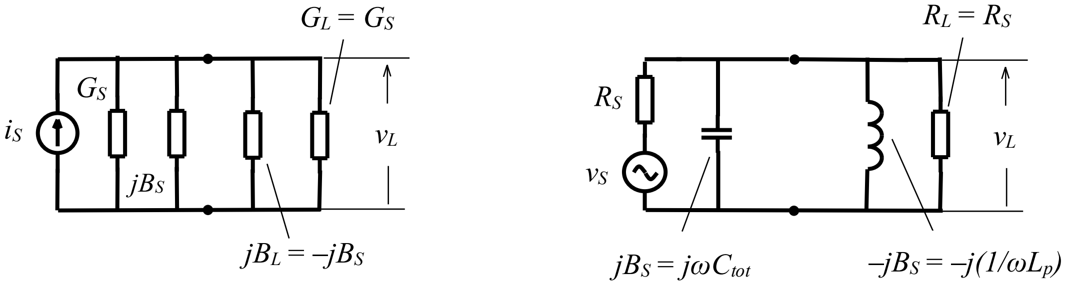
resonance circuits, the resonance curve is not symmetrical and the 3 dB bandwidth can be only approximately calculated with the conventional $B = f_0 / Q$ formula.

It must be noted that in the case of a modulated signal that has side-bands, the limited bandwidth corresponds to the attenuation of the side-frequencies, which leads to the deterioration of the transfer of information and reflection at these frequencies. The acceptable attenuation or the amount of reflection at the far ends of the side-bands must be evaluated according to the needs of the application.

Since the original matching theorem is stated in terms of the "impedances", the physical interpretation directly brings to mind the "series resonance". But it is also possible to express the matching relation in terms of the "admittances". It can be shown that for maximum power transfer from a signal source to its load, the load admittance must be equal to the complex conjugate of the source admittance, and vice versa (see Fig. 3). In this case the imaginary parts (susceptances) of the admittances can be cancelled out by "parallel resonance". Since the parasitic input and output admittances of electronic devices are usually represented as parallel capacitances connected to the input and output nodes, this approach lends itself more easily to achieve the matching condition that can be fulfilled by an inductance connected in parallel. It must be noted that for a parallel resonance the quality factor is:

$$Q = \frac{R_{par}}{L\omega_0}$$

Since the parallel resistance is equal to $R_S/2$, which is usually very low, it results in an extremely low Q-value.



(a) (b)

Fig. 3 (a) The principal diagram of parallel resonance matching. **(b)** Its practical application. C_{tot} is the sum of the parallel capacitances of the source side and the load side, L_p is the inductance necessary to resonate the total capacitance at the operation frequency.

Following these considerations, the matching problem can be expressed with an engineering approach as follows:

In a matched connection, for maximum power transfer from the signal source to its load, the existing parasitic series and/or parallel reactive component must be annulled using series and/or parallel resonances.

Example 1

For an on-chip matched inter-stage coupling at 10 GHz, the output impedance of the first stage is 50 ohms connected in parallel to 300 fF, and the input impedance of the following stage is 50 ohms connected in parallel to 120 fF, as shown in Fig. 4a. The variation of the signal voltage at the input of the following stage for this unmatched case is shown in Fig. 5a. To eliminate the effect of the total parallel capacitance at the coupling port, which is 420 fF, it must be parallel-resonated with an inductance at 10 GHz (see Fig. 4b). The calculated value of this inductance is 0.603 nH. The simulation result corresponding to this case is shown in Fig. 5.

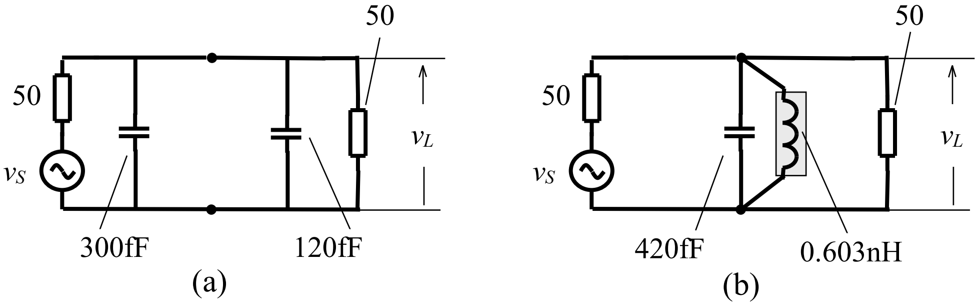


Fig. 4 (a) Original schematic of the inter-stage coupling. **(b)** Matching with parallel resonance at 10 GHz.

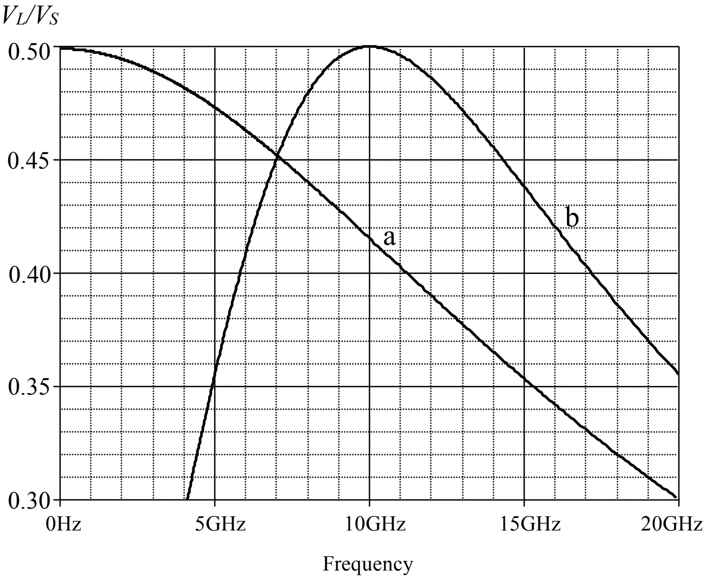


Fig. 5 Simulation results of (V_L/V_S) **(a)** non-matched coupling, **(b)** matched coupling. The voltage at the input of the second stage is half of V_S at resonance, as expected.

Example 2

A 50 ohm transmission line is connected to the input of an amplifier with a 4 mm-long, 25 µm-diameter bonding wire. The input impedance of the amplifier is 50 ohms connected in parallel to the 200 fF input capacitance. The center (carrier) frequency of the signal to be transferred is 5 GHz (see Fig. 6a). For maximum power transfer at 5 GHz, the inductance of the bonding wire, which is 4.57 nH, must be resonated in series at 5 GHz with a capacity of 222 fF. Additionally, the input capacitance of the amplifier must be resonated in parallel with an inductance of 5.07 nH (see Fig. 6b). The simulation result for the non-matched and matched cases are shown in Fig. 7.

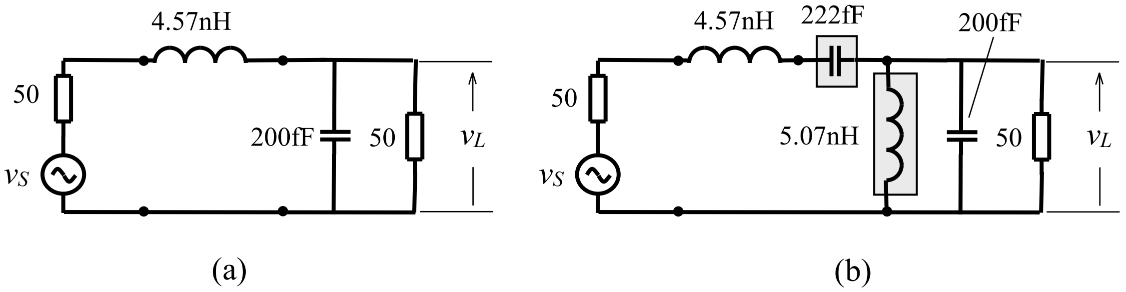


Fig. 6 Schematics of a **(a)** non-matched, **(b)** matched circuit

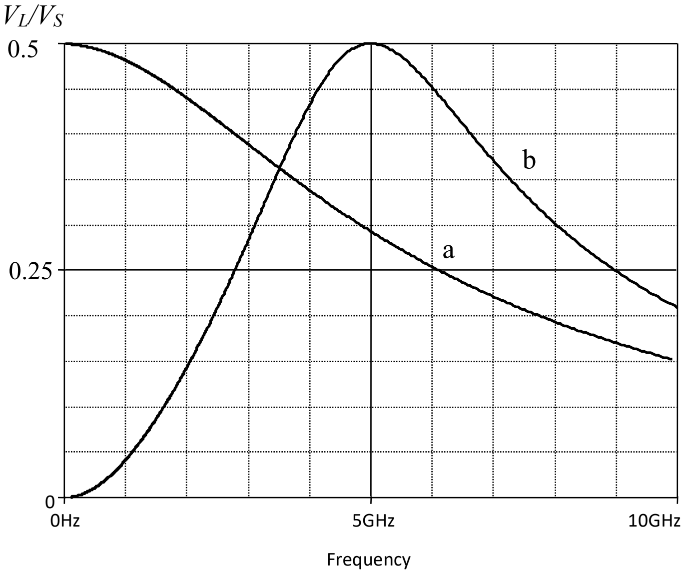
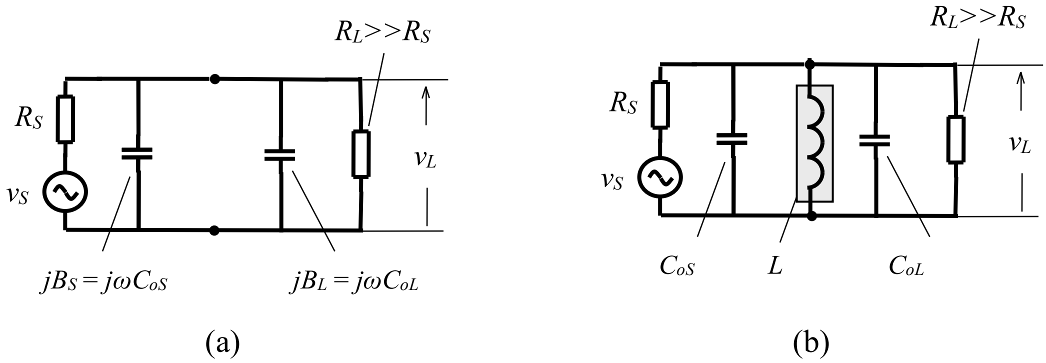


Fig. 7 Variation of (V_L/V_S) **(a)** before matching, **(b)** after matching

The concept explained above can be applied to wideband non-matched inter-stage couplings. It is known that, for this case, to minimize the effect of the parallel parasitic capacitance on the coupling port, the parallel resistance must be low. To realize this with a good voltage transfer efficiency, there are two possibilities. (i) A high internal resistance current source at the driving side and a low input resistance at the driven side, as in the Cherry-Hooper amplifier; (ii) a low internal resistance voltage source at the driving side and a high input resistance at the driven side, as shown in Fig. 8a.



(a) (b)

Fig. 8 (a) A typical non-matched inter-stage coupling. **(b)** To transfer a signal having a carrier frequency higher than the –3dB frequency of the R-C circuit, the total parallel capacitance has to be resonated with an appropriate inductance.

In Fig. 8a, the parallel resistance and the parallel capacitance on the connecting port are

$$R_p = \frac{R_S R_L}{R_S + R_L} \cong R_S, \quad C_p = C_{oS} + C_{oL}$$

respectively, and the –3dB frequency related to this low-pass port is $\omega_h = 1/R_p C_p$. To transfer a signal having a higher frequency with a high (unity) transfer efficiency, it is possible to resonate C_p with an appropriate parallel inductance. In this case, at resonance frequency, the voltage transfer efficiency of the port is unity, provided that $R_L \gg R_S$. The bandwidth depends on the quality factor of the parallel resonance circuit, which is $Q \cong R_S / L_p \omega_0$. Since R_S is small, it provides a high bandwidth suitable for many applications.

Fig. 9 shows the simulation results of a non-matched inter-stage coupling whose parameters are R_S = 50 ohms, C_{oS} = 100 fF, R_L = 10 kohms and C_{oL} = 50 fF. The –3dB frequency of this port is f_h = 21.22 GHz (see Fig. 9a). To transfer a 30 GHz signal over this coupling, the total parallel capacitance can be resonated at 30 GHz with an appropriate parallel inductance (0.188 nH in this case). The simulation results given in Fig. 9b show that the 30 GHz signal can be transferred with a unity voltage transfer ratio and with a –3dB bandwidth of approximately 20 GHz, and a –1dB bandwidth of 7 GHz.

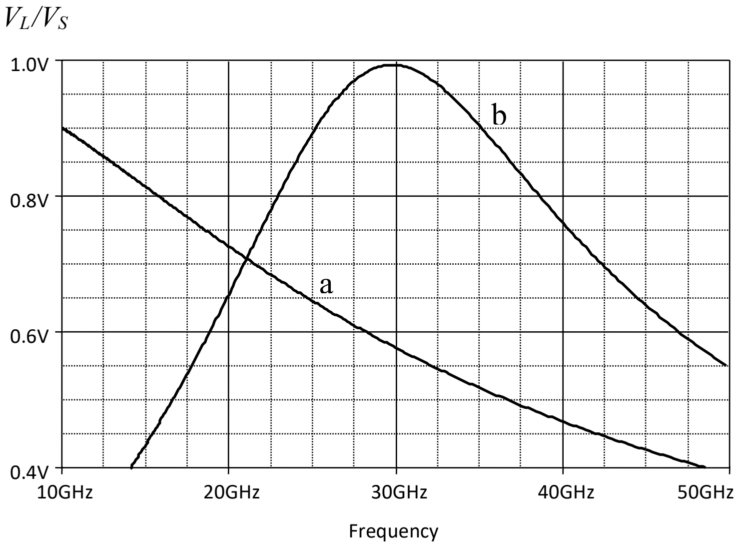


Fig. 9 The voltage transfer ratio of the non-matched inter-stage coupling **(a)** as is, **(b)** with the appropriate parallel inductance for 30 GHz.